Economics Letters 38 (1992) 435–441
North-Holland

International evidence on output persistence from postwar data

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Received 14 May 1991
Accepted 18 July 1991

This paper reports some international evidence on output persistence based on postwar data. Using fractional differencing analysis, we find evidence of fractional integration in output for the U.S. and U.K. but not for the other G-7 countries, implying that these two countries show much less persistence in output fluctuations than the others.

1. Introduction

The persistence of output fluctuations has been a hotly contested subject of research, important for evaluating theories of economic fluctuations. Based on long annual data for the United States, Nelson and Plosser (1982) report that the hypothesis of the presence of a unit root in real output cannot be rejected, and that shocks to real output are largely persistent. Campbell and Mankiw (1987) provide impulse response estimates of output persistence and show that shocks to output are not dissipated but amplified for the U.S. quarterly postwar data. In contrast, Watson (1986) and Clark (1987) analyze persistence in output using unobservable components models and find only a small permanent component. Clark (1989) reports similar output behavior for several other major industrialized countries. Cochrane (1988), employing the variance ratio approach, reports low output persistence for the U.S. Cogley (1990) notes that Cochrane's low persistence estimate is unique to the U.S. and does not apply to many OECD countries. Campbell and Mankiw (1989) present evidence of high persistence in output fluctuations for the U.S. and other G-7 countries.

Diebold and Rudebusch (1989) extend Campbell and Mankiw's (1987) impulse response analysis to fractionally integrated processes, discussed by Granger and Joyeux (1980) and Hosking (1981). By avoiding the knife-edged unit root and no unit root distinction, fractional time series models can provide better low-frequency approximations to the Wold representation than previous stochastic specifications such as the ARIMA and unobserved components models. The notion of fractional integration is of economic significance since a fractionally integrated process, unlike a unit root

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process, can be shown to display shock dissipation. Diebold and Rudebusch (1989) employ a two-stage semi-parametric procedure [Geweke and Porter-Hudak (1983)] to estimate fractional time series models for various U.S. output series. They find evidence of fractional integration and report that the effect of a shock to output will die out.

This study supplements previous evidence on output persistence by extending Diebold and Rudebusch's (1989) work in two respects. First, in this paper fractional time series models are estimated using a relatively new maximum likelihood (ML) procedure [Li and McLeod (1986) and Sowell (1990b)]. An advantage of the two-stage semi-parametric procedure employed in Diebold and Rudebusch (1989) is that it does not require an explicit parameterization of the ARMA part of the underlying process. However, a problem with the two-stage procedure is that little is known about its distributional properties. Further, the ML estimation makes full use of information in the data and can provide more precise estimates than the two-stage semi-parametric procedure. Second, this paper examines if the finding of fractional integration is common to other countries or unique to the U.S. based on postwar data on real output. Six other countries are considered; they include Canada, France, Germany, Italy, Japan and the U.K..

The paper is organized as follows. Section 2 examines the persistence of fractionally integrated processes in general. Section 3 outlines the empirical methodology of fractional differencing. Section 4 contains the empirical results. Section 5 concludes.

2. Fractional integration and persistence

A series is said to be integrated of order d, denoted by I(d), if it has a stationary, invertible ARMA representation after applying the differencing operator $(1-L)^d$. When d is not an integer the series is said to be fractionally integrated. A fractionally integrated process can be represented by

$$B(L)(1-L)^d x_t = D(L)u_t, \tag{1}$$

where $B(L) = 1 - \beta_1 L - \cdots - \beta_p L^p$, $D(L) = 1 + \delta_1 L + \cdots + \delta_q L^q$, all roots of $B(L)$ and $D(L)$ lie outside the unit circle, u_t is i.i.d. $(0, \sigma^2)$, and the fractional differencing operator given by

$$(1-L)^d = \sum_{k=0}^{\infty} \Gamma(k-d) L^k / \{\Gamma(k+1)\Gamma(-d)\}, \tag{2}$$

where $\Gamma(\cdot)$ is the Gamma function. Model (1) is referred to as the autoregressive fractionally integrated moving average (ARFIMA) model, which extends the standard ARIMA(p, d, q) model to real values of d. The extension raises the flexibility in modeling low-frequency dynamics by allowing for a richer class of spectral behavior at low frequencies than that implied by the ARIMA model. This can be seen from the spectral density of $x_t f_x(\lambda)$, which behaves like λ^{-2d} as $\lambda \to 0$ [Granger and Joyeux (1980) and Hosking (1981)]. For $d > 0$, $f_x(\lambda)$ is unbounded at frequency $\lambda = 0$, rather than bounded as for a stationary ARIMA(p, 0, q) series. By permitting d to take non-integer values, the ARFIMA model can thus capture a wide range of low-frequency behavior, not accommodated by conventional time series models.

The long-term persistence of the ARFIMA process x_t is determined by its order of integration, d. Specifically, it depends on whether d is less than unity or not. It is commonly known that the effect of a shock is long-lived for an I(1) process but can die out quickly for an I(0) process. Analytically, making the strict I(1) and I(0) distinction is not necessary for identifying shock dissipation, because a shock-dissipating process does not have to be I(0) exactly. A more general process such as an I(d) process with $d < 1$ can also exhibit shock dissipation. This can be seen from the moving average representation for $(1 - L)x_t$:

$$(1 - L)x_t = A(L)u_t, \tag{3}$$

where

$$A(L) = 1 + \alpha_1 L + \alpha_2 L^2 + \cdots,$$

derived from

$$A(L) = (1 - L)^{1-d}\Phi(L), \tag{4}$$

with $\Phi(L) = B^{-1}(L)D(L)$ and $\Phi(1) \neq 0$. The moving average coefficients α_i are called the impulse responses. The impact of a unit innovation at time t on the value of x at time $t + k$ is equal to $1 + \alpha_1 + \alpha_2 + \ldots + \alpha_k$. For shock dissipation, the infinite cumulative impulse response $A(1)$ equals zero, implying no long-run impact of the innovation on the value of x. Using eq. (2) to find the series representation for $(1 - L)^{1-d}$, eq. (4) can be written as

$$A(L) = F(d - 1, 1, 1; L)\Phi(L), \tag{5}$$

where $F(\cdot)$ is the hypergeometric function defined by

$$F(m, n, s; L) = \sum_{j=0}^{\infty} \Gamma(m + j)\Gamma(n + j)\Gamma(s)L^j / \{\Gamma(m)\Gamma(n)\Gamma(s + j)\Gamma(j + 1)\}. \tag{6}$$

Using some known properties of the hypergeometric function [e.g., Gradshteyn and Ryzhik (1980, pp. 1039–1042)], we can show that $F(d - 1, 1, 1; 1) = 0$ for $d < 1$, $F(d - 1, 1, 1; 1) = 1$ for $d = 1$, and $F(d - 1, 1, 1; 1) = \infty$ for $d > 1$. It follows that

$$\begin{aligned} A(1) = F(d - 1, 1, 1; 1)\Phi(1) &= 0 \quad &&\text{for} \quad d < 1, \\ &= \Phi(1) &&\text{for} \quad d = 1, \\ &= \infty &&\text{for} \quad d > 1. \end{aligned} \tag{7}$$

Hence, an I(d) process with $d < 1$ is shock-dissipating. For an I(1) process, $A(1)$ is finite and nonzero, so the effect of a shock will not die out. [1] For an I(d) process with $d > 1$, it will not display shock dissipation but shock amplification. Accordingly, a test for shock dissipation can be conducted as a test for fractional integration with order $d < 1$, without computing the value of $A(1)$ explicitly.

[1] In the case of a simple random walk, for example, $\Phi(1) = 1$ and so $A(1) = 1$; this implies that a 1 percent innovation to the variable will change its long-run forecast by exactly 1 percent.

3. Statistical analysis

The parameters of the ARFIMA model can be jointly estimated in time domain using the exact ML method, recently proposed by Li and McLeod (1986) and Sowell (1990b). The likelihood function of the ARFIMA model is given by

$$\mathscr{L}(Z\mid\cdot) = (2\pi)^{-T/2}\,|\Omega|^{-1/2}\exp(-Z'\Omega^{-1}Z/2) \tag{8}$$

where $Z = (x_1, x_2, \ldots, x_T)$, Ω is the $T \times T$ covariance matrix of Z. Li and Mcleod (1986) show that the ML estimators for d, β_i's and δ_i's, obtained by maximizing $\mathscr{L}(Z\mid\cdot)$ with respect to the parameters, are consistent and asymptotically normal. Note that Ω has T distinct elements $\{\gamma(k), k = 0, 1, \ldots, T-1\}$, where $\gamma(k)$ is the autocovariance at lag k. Sowell (1990b) shows that

$$\gamma(k) = \sum_{i=1}^{p}\zeta_i \sum_{n=0}^{p}\sum_{m=0}^{q}\beta_n\beta_m\Psi(d, d, p+n-m-k; r_i), \tag{9}$$

where r_i is the ith root of the autoregressive lag polynomial

$$\zeta_i = \left\{ r_i \prod_{j=1}^{p}(1-r_ir_j) \prod_{k=1,\neq i}^{p}(r_i - r_k)\right\}^{-1}, \tag{10}$$

and

$$G(h, j, k) = \Gamma(1-h-j)\Gamma(j+k)/\{\Gamma(1-h+k)\Gamma(1-j)\Gamma(j)\}, \tag{11}$$

$$\Psi(h, j, k; v) = G(h, j, k)\{v^{2p}F(j+k, 1, 1-h+k; v) + F(h-k, 1, 1-j-k; v) - 1\}, \tag{12}$$

with $F(\cdot)$ being the hypergeometric function defined before. The fact that the covariance matrix Ω is Toeplitz can be used to simplify the computation of Ω^{-1} and $|\Omega|$ in estimation.

For model identification the order of the ARFIMA model is selected using two information criteria, namely the Akaike information criterion (AIC) and the Schwarz information criterion (SIC). By the AIC and the SIC, one chooses the model that maximizes, respectively, $\ln\mathscr{L} - N$ and $\ln\mathscr{L} - N\ \ln T/2$, where $\mathscr{L}$ is the likelihood and N the number of the parameters estimated. Due to the difference in the adjustment term for the model dimension, the SIC tends to favor a lower-dimensional model than the AIC. To capture the short-run dynamics, ARFIMA(p, d, q) models with p and $q = 0, 1, 2, 3$ are considered, giving sixteen possible model specifications. The parameters of each model are estimated by the Davidson-Fletcher-Powell numerical optimization algorithm, and a convergence limit of 1×10^{-8} is imposed. Different starting values for parameters were used, and the final parameter estimates were generally not sensitive to the startup parameter values.

4. Empirical results

The data examined in this paper include quarterly data on real gross domestic product (GDP) and monthly data on industrial production (IP) for the postwar period obtained from the

Table 1
Estimated ARFIMA(p, d, q) models for quarterly data. [a]

Country	Criterion	$\{p, q\}$	$d-1$	β_1	β_2	δ_1	δ_2	δ_3
Canada	AIC	{0, 0}	0.182 (2.157)					
	SIC	{0, 0}	0.182 (2.157)					
France	AIC	{2, 0}	0.250 (2.549)	−0.694 (−5.521)	−0.293 (−2.590)			
	SIC	{2, 0}	0.250 (2.549)	−0.694 (−5.521)	−0.293 (−2.590)			
Germany	AIC	{2, 3}	0.137 (0.786)	−1.105 (−14.145)	−0.741 (−9.552)	−0.917 (−5.225)	0.449 (1.781)	−0.390 (−2.188)
	SIC	{0, 0}	−0.054 (−0.793)					
Italy	AIC	{0, 0}	0.079 (1.080)					
	SIC	{0, 0}	0.079 (1.080)					
Japan	AIC	{1, 0}	0.360 (4.984)	−0.392 (−4.048)				
	SIC	{1, 0}	0.360 (4.984)	−0.392 (-4.048)				
U.K.	AIC	{2, 2}	−0.130 (−1.65)	−0.179 (−1.871)	−0.803 (−10.108)	0.081 (1.513)	0.961 (19.725)	
	SIC	{0, 0}	−0.168 (−2.497)					
U.S.	AIC	{1, 0}	−0.554 (−3.187)	0.815 (6.676)				
	SIC	{1, 0}	−0.554 (−3.187)	0.815 (6.676)				

[a] The estimated ARFIMA model is given by $B(L)(1-L)^d x_t = D(L)u_t$. The t-statistics are given in parentheses.

International Monetary Fund's *International Financial Statistics* data tape. [2] The G-7 countries are considered, as in Campbell and Mankiw (1989) and Clark (1989); they are Canada, France, Germany, Italy, Japan, the U.K. and the U.S.. The quarterly data are collected from 1957:1 to 1989:4 for Canada, Japan, the U.K. and the U.S., from 1960:1 to 1989:4 for Italy and Germany, and from 1965:1 to 1989:4 for France. The monthly data are collected from 1957:1 to 1989:12 for Canada, France, Germany, the U.K. and the U.S., from 1959:1 to 1989:12 for Japan, and from 1961:1 to 1989:12 for Italy. All output series are expressed in natural logarithms.

As a preliminary data analysis, the augmented Dickey-Fuller (ADF) test for a unit root was conducted on individual GDP and IP series. Except for the case of Japan, the ADF test statistics indicated that the hypothesis of a unit root could not be rejected at the 5 percent level of

[2] In some cases where GDP is not available, gross national product (GNP) is then used.

significance for either the GDP or the IP series. When a linear trend was included in the test, in no case could the unit root hypothesis be rejected. In general, the GDP and IP series for the different countries seemed to follow an I(1) process. These results can say little, however, about possible fractional integration in the data because of the low power of the ADF test against fractional alternatives, as noted by Diebold and Rudebusch (1991) and Sowell (1990a). The ADF test, which is based on a strict I(1) and I(0) distinction, does not allow for the wider range of shock-dissipating behavior that captured is by fractionally integrated processes.

Table 1 contains the results of estimation of ARFIMA models for the quarterly GDP data. The numbers in parentheses are asymptotic t-statistics for the corresponding estimates of the model parameters. We observe that in only two cases, namely the cases of the U.K. and the U.S., the $d-1$ estimate is significantly less than zero. The associated p-value for the null hypothesis of $d=1$

Table 2
Estimated ARFIMA(p, d, q) models for monthly data. [a]

Country	Criterion	$\{p, q\}$	$d-1$	β_1	β_2	β_3	δ_1	δ_2	δ_3
Canada	AIC	{3, 2}	−0.027	1.469	−0.472	−0.137	−1.716	0.886	
			(−0.186)	(11.357)	(−3.152)	(−1.502)	(−22.094)	(9.403)	
	SIC	{0, 1}	0.227				−0.427		
			(1.344)				(−4.176)		
France	AIC	{2, 1}	−0.018	0.156	−0.200		−0.510		
			(−0.144)	(1.271)	(−2.972)		(−2.539)		
	SIC	{0, 2}	−0.009				−0.350	−0.228	
			(−0.083)				(−2.877)	(−4.636)	
Germany	AIC	{2, 0}	0.085	−0.660	−0.280				
			(1.537)	(−9.910)	(−4.621)				
	SIC	{2, 0}	0.085	−0.660	−0.280				
			(1.537)	(−9.910)	(−4.621)				
Italy	AIC	{0, 1}	−0.071				−0.575		
			(−0.698)				(−5.638)		
	SIC	{0, 1}	−0.071				−0.575		
			(0.698)				(−5.638)		
Japan	AIC	{2, 0}	0.628	−0.848	−0.454				
			(6.313)	(−14.008)	(8.105)				
	SIC	{2, 0}	0.628	−0.848	−0.454				
			(6.313)	(−14.008)	(−8.105)				
U.K.	AIC	{0, 0}	−0.419						
			(−3.638)						
	SIC	{0, 0}	−0.149						
			(−3.638)						
U.S.	AIC	{1, 0}	−0.434	0.849					
			(−4.304)	(12.975)					
	SIC	{1, 0}	−0.434	0.849					
			(−4.304)	(12.975)					

[a] The estimated ARFIMA model is given by $B(L)(1-L)^d x_t = D(L)u_t$. The t-statistics are given in parentheses.

against the one-sided alternative of $d < 1$ is less than 0.01 in either case. The qualitative results are not sensitive to whether the AIC or the SIC is used for model identification.

Table 2 contains the results from estimating ARFIMA models for the monthly IP data. Again, in both the U.K. and the U.S. cases the hypothesis of $d = 1$ can be rejected in favor of the alternative hypothesis of $d < 1$ at the 1 percent level of significance. In all the other five cases, however, we find no significant evidence of $d < 1$. Hence, the results for the IP data complement those for the GDP data in finding evidence of fractional integration in the output series for both the U.K. and the U.S. but not for the other G-7 countries.

5. Conclusion

In this paper some international evidence on the persistence of output fluctuations has been reported based on the postwar data for the G-7 countries. Both quarterly data on GDP and monthly data on industrial production are examined using fractional differencing analysis. The empirical results suggest that while the finding of fractional integration in output reported by Diebold and Rudebusch (1989) for the U.S. also holds for the U.K., it appears not common among the rest of the G-7 countries. The results imply that the U.S. and the U.K. show much less persistence in output fluctuations than the other G-7 countries.

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