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Long-run purchasing power parity during the recent float

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This paper examines the relevance of long-run purchasing power parity (PPP), which allows for measurement errors, during the recent floating exchange rate period. Previous empirical studies generally fail to find support for long-run PPP over this period. In this paper the cointegration property of exchange rates and prices is examined using a maximum likelihood procedure, and we find significant evidence favorable to long-run PPP. Further tests for symmetry and proportionality indicate that these two conditions are not generally consistent with the data. The results support the hypothesis of long-run PPP with measurement errors in prices.

1. Introduction

The purchasing power parity (PPP) theory in its 'absolute version' suggests that the long-run equilibrium exchange rate between two countries' currencies equals the ratio of their price levels. When the theory is applied to the real world, the relation between exchange rates and national price levels can be affected by many factors, however. The existence of non-traded goods and services, for example, can weaken the link between exchange rates and price levels. The link can also be weakened by transport costs, trade restrictions and imperfect competition. In addition, price-level measurements carry with them the usual problems associated with aggregation and index construction. International differences in consumption patterns, variations in product qualities, and differences between listed and transaction prices are some of the measurement problems which can affect the relation between exchange rates and price levels.

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Recent empirical studies mostly find less than supportive evidence for long-run PPP, especially since the return to floating exchange rates in the early 1970s. For example, Roll (1979), Frenkel (1981) and Adler and Lehmann (1983) report that the real exchange rate follows closely a random walk, suggesting little tendency for deviations from PPP to reverse. Huizinga (1987) finds some evidence of mean reversion in the real exchange rate, but not statistically significant. Baillie and Selover (1987), Corbae and Ouliaris (1988), Taylor (1988) and Mark (1990) fail to find cointegration between exchange rates and relative prices, implying that the two series tend to drift apart without bound. A general result of the above studies is that long-run PPP appears not to hold during the recent floating exchange rate period. Other studies by Frenkel (1978), Enders (1988), Taylor and McMahon (1988) and Grilli and Kaminsky (1989) note that PPP may perform better in other historical periods. Recently, Abuaf and Jorion (1990), Ardeni and Lubian (1991), Diebold, Husted and Rush (1991) and Glen (1991) studied long low-frequency (annual) data that encompass different historical periods of exchange rate systems, and these studies report evidence of mean reversion towards PPP. Nonetheless, an issue that still remains is the apparent empirical failure of long-run PPP during the recent period of floating exchange rates.

This study attempts to fill in the gap in the empirical support for long-run PPP. We find evidence favorable to long-run PPP in high-frequency (monthly) data for the recent floating exchange rate period. The empirical analysis is based on the theory of cointegration. This approach allows us to test for long-run PPP while abstracting from the short-run dynamics, and it can properly account for non-stationarity in the time series of exchange rates and prices. If PPP is true, deviations from a linear combination of exchange rates and prices should be stationary, i.e. exchange rates and prices should be cointegrated according to Engle and Granger (1987).

Previous cointegration analyses of long-run PPP are primarily based on residual-based tests following Engle and Granger's (1987) two-step procedure. This paper re-examines the validity of long-run PPP using a relatively new test for cointegration devised by Johansen (1991) in the context of vector autoregressions (VARs). Applications of the statistical technique have been illustrated by Johansen and Juselius (1990a, b). In contrast to the Engle–Granger procedure, Johansen's procedure takes into account the error structure of the data processes and allows for interactions in the determination of the relevant economic variables.

This paper addresses the issue of measurement errors in testing of long-run PPP. Taylor (1988) and Taylor and McMahon (1988) note that since observed price series are imperfect proxies at best for the theoretical price variables, the usual symmetry and proportionality restrictions under PPP are not necessarily consistent with empirical data. While these restrictions can be tested and not imposed in cointegration analysis, their validity is a maintained hypothesis in unit root tests for long-run PPP using real exchange rates. We observe that, if the symmetry and proportionality conditions are not consistent with the data, their imposition can bias PPP tests on real exchange rates towards finding no mean reversion. This is because a linear combination of non-stationary series, except with the correct cointegrating coefficients, is generally also non-stationary. It follows that failure to find reversion towards PPP can be due to violations of the two restrictions imposed a priori on the exchange rate and price data. In this paper we formally test for the validity of the two restrictions, and so for the measurement error hypothesis, in the cointegration framework.

2. Long-run PPP with measurement errors

For the purpose of empirical testing, the PPP relationship is written as

$$s_t = c + \alpha_1 p_t - \alpha_2 p_t^* + u_t, \tag{1}$$

where c is some constant, s_t is the logarithm of the spot exchange rate (domestic price of foreign currency), p_t and p_t^* are, respectively, the logarithms of the domestic and foreign price indices, and u_t is an error term capturing deviations from PPP. For PPP to hold in the long run, u_t should be stationary. While symmetry between the domestic and foreign countries requires that $\alpha_1 = \alpha_2$, the long-run proportionality between exchange rates and prices implies $\alpha_1 = 1 = \alpha_2$.

The general specification of PPP can be motivated by the presence of measurement errors in prices [Taylor (1988)]. Suppose that long-run PPP holds for some theoretical price indices, denoted by g_t and g_t^* so that

$$s_t = h + g_t - g_t^* + d_t, \tag{2}$$

where d_t is a stationary process. Since theoretical price variables are not observable, they are proxied by some observed price indices in empirical analysis. Depending on how the observed indices are constructed, their constituent elements and the sources of price changes, a 1 percent change in the observed price indices can correspond to a more than or less than 1 percent change in their theoretical counterparts. This can be captured by considering the observed price series, p_t and p_t^* to be related to the theoretical ones by the following measurement equations:

$$p_t = a_1 + b_1 g_t + \varepsilon_{1t}, \tag{3}$$

$$p_t^* = a_2 + b_2 g_t^* + \varepsilon_{2t}, \tag{4}$$

where the parameters a_1, a_2, b_1, and b_2 capture systematic measurement errors; ε_{1t} and ε_{2t} are stationary stochastic terms capturing non-systematic measurement errors. The stationarity in ε_{1t} and ε_{2t} implies that the observed prices will not drift too far apart from the theoretical prices, which is a requirement for a meaningful test of PPP. The parameters b_1 and b_2 can differ from one another due to differences between countries in the composition of goods and services and in the weighting scheme used for index construction. Combining eqs. (2), (3) and (4) yields eq. (1), with $c = h - a_1/b_1 + a_2/b_2$, $\alpha_1 = 1/b_1$, $\alpha_2 = 1/b_2$ and $u_t = d_t - \varepsilon_{1t}/b_1 + \varepsilon_{2t}/b_2$. Since in general either α_1 or α_2 or both can differ from unity, eq. (1), in contrast to eq. (2), can be viewed as a PPP relationship with measurement errors in prices.

Relationship (1) can in general be expressed as

$$\alpha' X_t = c + u_t, \tag{5}$$

where X_t is a vector series given by $(s_t, p_t, p_t^*)'$ and $\alpha = (1, -\alpha_1, \alpha_2)'$. When the series in X_t are I(1), i.e. integrated of order one, u_t is generally also I(1). However, if there exists an α such that u_t is stationary or I(0), as implied by long-run PPP, X_t is said to be cointegrated and $\alpha' X_t$ represents a long-run equilibrium relationship [Engle and Granger (1987)]. A test for long-run PPP can thus be undertaken based on the theory of cointegration.

3. Testing for cointegration

The Johansen (1991) test is based on the technique of reduced rank regression. Consider in general an $n \times 1$ time-series vector $X_t = (x_{1t}, \ldots, x_{nt})'$. Regress ΔX_t on a constant and $\Delta X_{t-1}, \ldots, \Delta X_{t-k}$, giving the residual $\hat{u}_{1t}$. Regress X_{t-k-1} on a constant and $\Delta X_{t-1}, \ldots, \Delta X_{t-k}$, giving the residual $\hat{u}_{2t}$. Define the product moment matrices of the residuals as $S_{ij} = T^{-1} \sum_{t=1}^{T} \hat{u}_{it} \hat{u}'_{jt}$, $i,j = 1, 2$. The likelihood ratio (LR) test statistic for the hypothesis of at most r cointegrating vectors is

$$-2 \ln Q_r = -T \sum_{j=r+1}^{n} \ln(1 - \phi_j), \tag{6}$$

where $\phi_{r+1}, \ldots, \phi_n$ are the $n-r$ smallest eigenvalues of $S_{21} S_{11}{}^{-1} S_{12}$ with respect to S_{22}. Critical values for the test are tabulated in Johansen and Juselius (1990a). Cheung and Lai (1991) examine the finite sample properties of the Johansen test and report that for a sample size of 200 or above, the empirical size of the test is rather close to its nominal size.

The maximum likelihood (ML) estimate of the cointegrating vector α can be found as the eigenvector $V = (v_1, \ldots, v_r)$, with the normalization that

$V'S_{22}V=I$. Phillips (1991) shows that the ML coefficient estimator is super-consistent, symmetrically distributed and median unbiased asymptotically. A Monte Carlo study by Gonzalo (1989) indicates that the finite sample properties of the ML estimator are largely consistent with the asymptotic results. This is even so when the errors are non-Gaussian or when the estimated model is overparameterized in lags. Monte Carlo evidence reported by Stock and Watson (1991) also confirms the unbiasedness property of the ML estimator, but in one of the simulation models its empirical distribution shows a larger dispersion than that of other estimators in small samples of size 100.

3.1. A power comparison of cointegration tests

The power properties of the Johansen test relative to standard residual-based tests are examined using the Monte Carlo method. The simulation experiment conducted and its results are presented in the appendix. The results indicate that both the augmented Dickey–Fuller (1979) or ADF test and the Phillips (1987) Z_α test have rather low power against interesting local alternatives. Using a 5 percent level of significance and for an autoregressive parameter $\rho=0.90$, the ADF and Z_α tests can reject the false null hypothesis of no cointegration about only 5 percent and 8 percent of the time, respectively. In contrast, the Johansen test appears to perform relatively well. Using the same 5 percent level of significance, the Johansen test is able to reject the false null of no cointegration about 24 percent of the time. Note that for monthly data and a value of the autoregressive coefficient ρ equal to 0.90, it takes about 6.6 months (13.2 months) to reduce a given deviation from equilibrium by 50 percent (75 percent), and the implied speed of adjustment seems not too slow. Hence, the residual-based tests have very low power in rejecting the no cointegration hypothesis even when an equilibrium relationship in fact holds in the long run with a reasonable speed at which deviations from equilibrium are corrected.

While the above Monte Carlo evidence is admittedly limited and alternative model specifications can be explored, the results serve as an illustration that there is a potentially significant power advantage of the Johansen test for cointegration over the standard residual-based tests. The results may reflect the fact that the Johansen procedure, which is based on full system estimation, can eliminate the simultaneous equation bias and raise efficiency relative to single equation methods [see for example, Gonzalo (1989); and Phillips (1991)].

3.2. The data and empirical results

The data examined are monthly series taken from the IMF's International

Financial Statistics data tape. The series include period averages of spot exchange rates in U.S. dollars (line ah) as well as price levels, measured alternatively by consumer price indices (CPIs) (line 64) and wholesale price indices (WPIs) (line 63). Five bilateral intercountry relations are considered between the United States as the home country and the United Kingdom, France, Germany, Switzerland and Canada as the foreign countries. The data sample covers the period from January 1974 to December 1989, except for the WPI series for France that runs from January 1974 to December 1986. All the data series are seasonally unadjusted, so the potential problem concerning distortionary effects of seasonal adjustments on unit roots tests [Ghysels (1990)] can be ignored.

The series of nominal exchange rates and price indices were each first checked for a unit root using the ADF and Z_α tests with a drift. According to the Z_α test, for all the series examined the hypothesis of a unit root could not be rejected at the 5 percent significance level. The ADF test statistics largely confirmed these results, except for the U.K. CPI and WPI series, which appear stationary. When a linear trend was included in the tests, in no case could the I(1) hypotheis be rejected at the 5 percent level. Unit root tests were applied also to the first-differenced series, and the I(1) null hypothesis could be rejected for all the series using the Z_α test. The ADF test yielded similar results, except or the French CPI series. These findings, in accordance with previous findings, suggest that the levels of exchange rates and prices are not stationary.

Table 1 contains the results of residual-based tests for cointegration. The tests examine stationarity of the residual from the cointegrating regression that normalizes on the exchange rate. As shown in table 1, the no cointegration null cannot be rejected in any case at the 5 percent level for both the ADF and Z_α tests. Similar results apply to the cases where symmetry is imposed in the cointegrating regressions. Hence, as reported in previous empirical studies, the residual-based tests consistently find little evidence of cointegration between nominal exchange rates and prices. Further tests on real exchange rates were conducted by imposing the proportionality condition, and in no case could the hypothesis of a unit root be rejected at the standard significance levels.

The Johansen test is next performed in the VAR framework. Different values of the lag length $k=1$ to 10 were considered. In most cases a lag of $k=8$ is required to remove serial correlation in the residuals, so statistical results based on a VAR(8) model are reported. Jarque and Bera's (1980) misspecification tests were conducted. The results [reported in Cheung and Lai (1990)] in general seem satisfactory. In the majority of the equations, especially exchange rate equations, the residuals passed the tests for being uncorrelated and normally distributed.

Table 2(a) reports the values of the Johansen test statistic, $-2\ln Q_r$, for the

Table 1

Results of residual-based tests for cointegration.

Country	CPI				WPI			
	α_1	α_2	ADF	Z_α	α_1	α_2	ADF	Z_α
U.K.	0.360	0.528	−1.894	−6.264	0.836	0.789	−2.021	−6.441
France	3.326	2.965	−1.976	−7.044	1.175	1.723	−1.978	−9.897
Germany	4.158	7.611	−2.720	−10.583	2.287	3.457	−2.022	−6.905
Switzerland	1.715	2.604	−1.942	−7.416	1.159	1.893	−1.981	−7.241
Canada	0.301	0.011	−0.392	−2.013	0.310	0.548	−0.654	−2.675
Symmetry imposed:								
U.K.	0.880	0.880	−1.935	−6.364	0.732	0.732	−2.008	−6.476
France	1.967	1.967	−1.810	−6.625	2.096	2.096	−3.028	−5.089
Germany	0.371	0.371	−1.447	−4.946	0.721	0.721	−1.505	−4.890
Switzerland	0.916	0.916	−1.979	−7.452	0.858	0.858	−1.985	−7.211
Canada	0.757	0.757	−0.885	−2.641	1.202	1.202	−1.143	−2.576

Notes: Critical values for the ADF test are given by −3.24 (5%) and −3.51 (2.5%) for $n=2$, and by −3.62 (5%) and −3.86 (2.5%). The lag used in the ADF test in each case is selected based on the Schwartz information criterion. Critical values for the Z_α test are given by −21.10 (5%) and −24.87 (2.5%) for $n=2$, and by −26.16 (5%) and −29.35 (2.5%) for $n=3$. The lag truncation parameter applied in the Z_α test is equal to 8. The above critical values are obtained based on the Monte Carlo method in 20,000 replications with $T=200$, assuming that the true system is of independent random walk processes that are not cointegrated.

hypothesis of at most r linearly independent cointegrating vectors. The Johansen test results differ dramatically from those of the residual-based tests. In all ten cases under consideration the hypothesis of no cointegrating vector ($r=0$) can be rejected at the 5 percent level, indicating that the series in X_t are cointegrated, as suggested by long-run PPP. To check whether or not the exchange rate enters into the cointegrating relationship, a LR test of a zero restriction on the cointegrating vector was performed. In all cases but one (the case of the Canadian CPI data) the hypothesis that the cointegrating relationship includes the exchange rate variable was accepted at the 5 percent significance level. Bilateral relations using countries other than the United States as the home country were tested. There were all together 15 possible pairwise combinations of the six countries examined. Significant evidence for cointegration could be found in 13 (11) out of the 15 cases where WPIs (CPIs) were used. The results are supportive of the long-run PPP hypothesis.

As noted earlier, the symmetry and proportionality restrictions need not hold empirically in the presence of measurement errors in prices. A test of either restriction can thus serve as a test of the significance of measurement errors. Rejections of either restriction, together with the above findings of cointegration, can illustrate that the failure to find cointegration in previous PPP analyses can be partly due to the low power of standard cointegration

Table 2
Results of the Johansen test.

(a) Testing for cointegration

	$-2\ln Q_r$					
	CPI			WPI		
Country H_0:	$r\leqq 2$	$r\leqq 1$	$r=0$	$r\leqq 2$	$r\leqq 1$	$r=0$
U.K.	5.341	23.602**	52.795**	9.531	31.834**	58.189**
France	4.888	17.297	44.002**	1.095	10.180	55.799**
Germany	4.290	15.134	50.686**	1.338	12.120	51.613**
Switzerland	1.518	6.416	33.159*	0.572	19.770**	59.055**
Canada	0.426	5.059	32.535*	1.282	8.231	34.070**

(b) Testing for symmetry and proportionality

	$-2\ln Q_G$							
	CPI				WPI			
	α_1	α_2	H_{sym}	H_{pro}	α_1	α_2	H_{sym}	H_{pro}
U.K.	1.426	1.035	8.946**	14.491**	0.350	0.933	2.435	5.368
France	3.826	3.521	0.592	9.506**	0.776	1.177	21.536**	26.384**
Germany	7.636	14.972	23.608**	24.543**	3.113	4.460	20.204**	20.562**
Switzerland	1.445	5.079	12.973**	15.250**	8.235	11.414	25.884**	26.907**
Canada	25.422	23.714	4.946*	25.805**	1.636	1.741	1.045	18.619**

Notes: (a) Critical values for the likelihood ratio statistic, $-2\ln Q_r$ $(0\leqq r\leqq n)$, are based on the simulated values tabulated in Johansen and Juselius (1990a, table A.2, p. 208). For $n-r=1$, the critical values are given by 8.083 (5%) and 9.658 (2.5%). For $n-r=2$, the critical values are given by 17.844 (5%) and 19.611 (2.5%). For $n-r=3$, the critical values are given by 31.256 (5%) and 34.062 (2.5%). Significance is indicated by * at the 5% level or ** at the 2.5% level.

(b) The $-2\ln Q_G$ statistics for testing the hypotheses of symmetry (H_{sym}) and proportionality (H_{pro}) have a chi-square distribution of, respectively, $r(n-2)$ and $r(n-1)$ degrees of freedom. Significance is indicated by * at the 5% level or ** at the 2.5% level.

tests and partly due to the imposition of incorrect coefficient restrictions on the cointegrating vector.

The hypotheses of symmetry and proportionality represent two testable linear restrictions on the cointegrating vector using a chi-square test developed by Johansen (1991) [see also Johansen and Juselius (1990a, b)]. Table 2(b) contains the results of the coefficient restriction tests. In seven cases (four for CPIs and three for WPIs) both restrictions can be rejected at the 5 percent level. In two additional cases (one for CPIs and one for WPIs) the proportionality restriction, though not the symmetry restriction, can be rejected at the 5 percent level. Hence, in nine out of the ten cases the restriction of either symmetry or proportionality is rejected statistically by the data.

The rejections of the symmetry and proportionality conditions highlight the desirability of working with a general unrestricted trivariate model in testing for cointegration and long-run PPP. When the symmetry or propor-

Table 3

Results of cointegration tests in bivariate or univariate models.

		$n=2$				$n=1$	
		CPI		WPI		CPI	WPI
Country	H_0:	$r \leq 1$	$r=0$	$r \leq 1$	$r=0$	$r=0$	$r=0$
U.K.		3.506	27.554**	4.632	33.811**	2.733	3.238
France		2.894	31.020**	3.038	11.115	1.186	1.112
Germany		0.501	16.930	2.013	22.271**	0.698	1.062
Switzerland		3.609	18.514*	3.425	20.810**	1.883	1.541
Canada		1.528	7.321	6.081	14.541	0.277	0.707

Notes: Critical values for the likelihood ratio statistic, $-2\ln Q_r(0 \leq r < n)$, are based on the simulated values tabulated in Johansen and Juselius (1990a, p. 208). For $n-r=1$, the critical values are given by 8.083 (5%) and 9.658 (2.5%). For $n-r=2$, the critical values are given by 17.844 (5%) and 19.611 (2.5%). Significance is indicated by * at the 5% level or ** at the 2.5% level.

tionality restriction is not supported by the data, its imposition, which leads to a bivariate or univariate model, can bias the test towards finding no cointegration. The point is that a linear combination of I(1) series, except the correct cointegrating relationship, is generally not stationary. As a result, the findings of no cointegration can be interpreted as rejections of the imposed restriction on the underlying equilibrium relationship rather than of the equilibrium condition itself. In addition, the symmetry and proportionality conditions are restrictions on the long-run coefficients only. When we reduce the trivariate model for (s, p, p^*) to the bivariate one for $(s, p-p^*)$ or the univariate one for $(s-p+p^*)$, restrictions on the short-run coefficients have to be imposed in addition to those on the long-run coefficients. The restricted models thus ignore any possible interactions in the determination of prices and exchange rates that are allowed for in the unrestricted trivariate model. We owe this point to Søren Johansen [see Cheung and Lai (1990) for more discussion].

To illustrate the possible differences in test results among trivariate, bivariate and univariate models, Johansen tests are conducted on models for $X_t=(s_t, p_t-p_t^*)'$ and for $X_t=(s_t-p_t+p_t^*)'$. Table 3 contains the statistical results for the bivariate (exchange rates and relative prices) and univariate (real exchange rates) models. For the bivariate models evidence of cointegration can be found in six out of ten cases at the 5 percent level or better. These results are not as favorable to long-run PPP as those based on the trivariate models. The results for the univariate models are particularly disappointing: in no case can we find supportive evidence for long-run PPP.

The foregoing results provide a cautionary note on interpreting the results of tests for long-run PPP based on the time-series behavior of the real exchange rate. Findings of non-stationarity in the real exchange rate can indicate a violation of the symmetry or proportionality restriction and still be consistent with long-run PPP with measurement errors. Several authors [e.g. Roll (1979); Adler and Lehmann (1983); Darby (1983) and Hakkio (1986)], note that the real exchange rate can or should follow a random walk, theoretically. The random walk proposition is based on a number of empirically questionable assumptions, e.g. interest rate parity holds, the forward rate is an unbiased predictor of the future spot rate, the Fisher relationship holds and real interest rate are constant. However, it seems difficult to reject this proposition empirically. Hakkio (1986) observes that this can be due partly to the low power of standard unit root tests. The present analysis indicates that the presence of measurement errors can also play a part in biasing the tests towards finding negative results.

4. Concluding remarks

Arguing that it is difficult to detect long-run PPP using high-frequency data, several recent studies, e.g. Abuaf and Jorion (1990), Ardeni and Lubian (1991) and Diebold, Husted and Rush (1991), examine chronologically long low-frequency (annual) data. These studies employ different econometric techniques, including multivariate unit root tests, variance ratio tests and fractional differencing, and find some evidence in favor of long-run PPP over different historical periods of exchange rate arrangement. Glen (1991), using variance ratio tests, examines both monthly post-Bretton Woods data and long historical annual data, and finds that the random walk hypothesis for real exchange rates can be rejected by either data set, but for different reasons. For the monthly data the rejection is explained by positive serial correlation rather than mean reversion, whereas for the annual data it is due to mean reversion. The present paper employs cointegration techniques and provides evidence for long-run PPP during the post-Bretton Woods period based on monthly data. It is found that some hypotheses maintained in previous work, namely symmetry, proportionality and no measurement error in prices, can lead to different results concerning the validity of PPP in the post-Bretton Woods period.

Appendix

To illustrate the difference in power between the residual-based tests and the Johansen test for cointegration, a simple Monte Carlo experiment, similar to that considered by Engle and Granger (1987), is conducted. A trivariate system of x_{1t}, x_{2t}, and x_{3t} is generated by

$$-x_{1t} - 3x_{2t} + 2x_{3t} = u_{1t}, \quad u_{1t} = u_{1t-1} + e_{1t}, \tag{A1}$$

$$4x_{1t} + x_{2t} - 2x_{3t} = u_{2t}, \quad u_{2t} = u_{2t-1} + e_{2t}, \tag{A2}$$

Table A.1.

ρ	Level of significance (%)	Rejections per 100 ($T=200$) ADF	Z_α	Johansen
0.90	5	5.2	7.8	23.9
0.80	5	9.3	25.4	56.8
0.70	5	15.5	34.4	81.2

$$x_{1t}-x_{2t}+x_{3t}=u_{3t}, \quad u_{3t}=\rho u_{3t-1}+e_{3t}, \tag{A3}$$

with e_{1t}, e_{2t} and e_{3t} being independent standard normal variates. It can be shown that when $|\rho|<1, x_{1t}, x_{2t}$ and x_{3t} are cointegrated and (A.3) is their cointegrating relationship. When $\rho=1$, however, the three series are not cointegrated. In the experiment samples of size $T=200$ are used. Sample series of x_1, x_2 and x_3 are generated by setting the initial values of u_1, u_2 and u_3 equal to zero and creating 250 observations, the first 50 of which are discarded to reduce the effect of the initial condition. Estimates of the powers of all tests are based on 1,000 replications, and the lag parameter for each test is chosen to be equal to 4. The simulation results are reported in table A.1.

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